

Marietta City Schools

2023–2024 District Unit Planner

Algebra: Concepts & Connections

Unit title Unit 4: Modeling and Analyzing Quadratic Functions MYP year 4 Unit duration (hrs) 27 hours

Mastering Content and Skills through INQUIRY (Establishing the purpose of the Unit): What will students learn?

GA DoE Standards

Standards

- **8.FGR.5*** Describe the properties of functions to define, evaluate, and compare relationships, and use functions and graphs of functions to model and explain real phenomena. **(teach prior to A.PAR.6)**
 - 8.FGR.5.1 Show and explain that a function is a rule that assigns to each input exactly one output.
 - **8.FGR.5.2** Within realistic situations, identify and describe examples of functions that are linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
 - **8.FGR.5.3** Relate the domain of a linear function to its graph and where applicable to the quantitative relationship it describes.
 - **8.FGR.5.4** Compare properties (rate of change and initial value) of two functions used to model an authentic situation each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
 - **8.FGR.5.5** Write and explain the equations y = mx + b (slope-intercept form), Ax + By = C (standard form), and $(y y_1) = m(x x_1)$ (point-slope form) as defining a linear function whose graph is a straight line to reveal and explain different properties of the function.
 - **8.FGR.5.6** Write a linear function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - **8.FGR.5.7** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
 - **8.FGR.5.8** Explain the meaning of the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
 - **8.FGR.5.9** Graph and analyze linear functions expressed in various algebraic forms and show key characteristics of the graph to describe applicable situations.
- **A.PAR.6:** Build quadratic expressions and equations to represent and model real-life phenomena; solve quadratic equations in contextual situations.
 - **A.PAR.6.1** Interpret quadratic expressions and parts of a quadratic expression that represent a quantity in terms of its context.

Fundamentals

- Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context.
- Given mathematically applicable situations which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning of given individual terms or factors
- **A.PAR.6.2** Fluently choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the expression.

Fundamentals

- Students should be able to multiply variable expressions involving the product of a monomial and a binomial and the product of two binomials to produce a quadratic expression.
- Polynomial operations are included with this objective. Polynomial sums, differences, and products should not exceed a maximum degree of 2.

Strategies and Methods

- Students should be able to move fluently (flexibly, accurately, efficiently) between different forms of a quadratic expression (standard, vertex, and factored forms).
- Students should be able to use the structure of a quadratic expression to rewrite it in different equivalent forms.
- **A.PAR.6.3** Create and solve quadratic equations in one variable and explain the solution in the framework of applicable phenomena.

Fundamentals

• Students should be able to multiply variable expressions involving the product of a monomial and a binomial and the product of two binomials to solve a quadratic equation.

Strategies and Methods

- Students should be able to solve quadratic equations fluently (flexibly, accurately, efficiently) by inspection, taking square roots, factoring, completing the square, and applying the quadratic formula, as appropriate to the initial form of the equation.
- Students should be able to fluently transform a quadratic equation in x into an equation of the form $(x p)^2 = q$ that has the same solutions.
- Students should be able to analyze and explain what the zeros describe in context.

Relevance and Application • Limit to real number solutions

A.PAR.6.4 Represent constraints by quadratic equations and interpret data points as possible or not possible in a modeling framework.

Terminology

• Possible data points are solutions to the equation(s); data points that are not possible are non-solutions to the equation(s)

A.FGR.7: Construct and interpret quadratic functions from data points to model and explain real-life phenomena; describe key characteristics of the graph of a quadratic function to explain a contextual situation for which the graph serves as a model.

A.FGR.7.1 Use function notation to build and evaluate quadratic functions for inputs in their domains and interpret statements that use function notation in terms of a given framework.

Fundamentals

- Students should apply their understanding of function notation from their work with linear functions to build, evaluate, and interpret quadratic functions using function notation.
- Students should be

A.FGR.7.2 Identify the effect on the graph generated by a quadratic function when replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

Strategies and Methods

• Students should be given opportunities to experiment with cases and illustrate an explanation of the effects on the graph using technology.

A.FGR.7.3 Graph and analyze the key characteristics of quadratic functions.

Strategies and Methods

• Students should be able to use verbal descriptions, tables, and graphs created using interactive technology tools.

Fundamentals

- Students should be able to sketch a graph showing key features including domain, range, and intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; asymptotes; end behavior.
- Key characteristics of the quadratic functions should be expressed in interval and set-builder notation using inequalities.

A.FGR.7.4 Relate the domain and range of a quadratic function to its graph and, where applicable, to the quantitative relationship it describes.

Examples

• If the function h(t) gives the path of a projectile over time, t, then the set of non-negative real numbers would be an appropriate domain for the function because time does not include negative values.

• A bird is building a nest in a tree 36 feet above the ground. The bird drops a stick from the nest. The function $f(x) = -16x^2 + 36$ describes the height of the stick in feet after x seconds. Graph this function. Identify the domain and range of this function. (A student should be able to determine that the appropriate values for the domain and range of this graph are $0 \le x \le 1.5$ and $0 \le y \le 36$, respectively.)

A.FGR.7.5 Rewrite a quadratic function representing a mathematically applicable situation to reveal the maximum or minimum value of the function it defines. Explain what the value describes in context.

Fundamentals

• Students should be able to interpret the maximum and minimum value of a quadratic function expressed in a variety of ways.

Strategies and Methods

• Students should be able to use interactive graphing technologies to make sense of the maximum and minimum values in context.

Example

• Consider the path of a football thrown through the air. When does the football reach its maximum height? How high does the football reach?

A.FGR.7.6 Create quadratic functions in two variables to represent relationships between quantities; graph quadratic functions on the coordinate axes with labels and scales.

Strategies and Methods

• Students should be able to use interactive graphing technologies to make sense of the visual, graphical model for a quadratic function representing a mathematically applicable situation.

A.FGR.7.7 Estimate, calculate, and interpret the average rate of change of a quadratic function and make comparisons to the average rate of change of linear functions.

Fundamentals

- Students should be given opportunities to estimate the rate of change from a graph.
- Students should be able to show that linear functions grow by equal differences over equal intervals and recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Students should be able to compare this behavior to that of the average rate of change of quadratic functions. This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals

Strategies and Methods

• Functions can be presented symbolically, as a graph, or as a table.

A.FGR.7.8 Write a function defined by a quadratic expression in different but equivalent forms to reveal and explain different properties of the function.

Strategies and Methods

• Students should be able to move fluently (flexibly, accurately, efficiently) between the factored form, vertex form, and standard form of a quadratic function.

Fundamentals

• Students should be able to examine a quadratic function by analyzing the zeros, extreme values, and symmetry of the graph and interpret these properties in context.

Strategies and Methods

• Students should be given opportunities to use a variety of strategies and methods to make sense of the properties of quadratic functions:

o Factoring o Completing the square o Quadratic formula o Graphing o Taking square roots

Example

• Students should be able to compare quadratic functions in standard, vertex, and intercept forms

A.FGR.7.9 Compare characteristics of two functions each represented in a different way.

Fundamentals

- Functions can be presented numerically in tables, algebraically, graphically, and by verbal descriptions.
- Students should be able to:
 - o compare a quadratic function to a linear function, or another quadratic function.
 - o compare key characteristics of quadratic functions with the key characteristics of linear functions.
 - o observe using graphs and tables that a quantity increasing quadratically will eventually exceed a portion of a quantity increasing linearly.

Examples

- Given a graph of one quadratic function and an algebraic equation for another, students should be able to determine which has the larger maximum.
- Given a graph of one function and an algebraic equation for another, students should be able to determine which has the larger y-intercept.

A.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics

A.MM.1.1 Explain applicable, mathematical problems using a mathematical model.

Fundamentals

- Students should be provided with opportunities to learn mathematics in the framework of real-life problems.
- Mathematically applicable problems are those presented in which the given framework makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).

A.MM.1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities domains.

Fundamentals

• Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.

A.MM.1.4 Use various mathematical representations and structures with this information to represent and solve real-life problems.

Strategies and Methods

- Students should be able to fluently navigate between mathematical representations that are presented numerically, algebraically, and graphically.
- For graphical representations, students should be given opportunities to analyze graphs using interactive graphing technologies.

A.MM.1.5 Define appropriate quantities for the purpose of descriptive modeling.

Fundamentals

• Given a situation, framework, or problem, students should be able to determine, identify, and use appropriate quantities for representing the situation.

Concepts/Skills to support mastery of standards

- 1. Use Function Notation
- 2. Put data into tables
- 3. Graph data from tables
- 4. Solve one variable linear equations
- 5. Determine domain of a problem situation
- 6. Solve for any variable in a multi-variable equation
- 7. Recognize slope of a linear function as a rate of change
- 8. Graph linear functions
- 9. Graph inequalities
- 10. Distinguish between linear and nonlinear functions

<u>Vocabu</u>	lary
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Axis of Symmetry	Completing the Square	Concavity	Decreasing	Degree	Difference of Two Squares
Discriminant	Function	Horizontal Shift	Increasing	Leading Coefficient	Maximum
Minimum	Parabolic	Perfect Square Trinomial	Quadratic	Quadratic Equation	Quadratic Expression
Quadratic Function	Representation	Root	Standard Form	Vertex	Vertex Form
Vertical Shift	Zeros				

Notation

Function Notation - f(t) Interval Notation - [.], (,) Set Notation - $D: \{x \mid x\}$

Set Notation - $D: \{x \mid x \in R\}$ (Set of all real numbers), $R: \{y \mid y \in R\}, \{x \mid 5 \le x \le 7\}$

Key concept	Related concept(s)	Global context
Relationships - Identify and understand connections and associations between properties, objects, people and ideas - including the human community's connections with the world in which we live.	Representation, Systems, and Models	Scientific and Technical Innovation - Mathematical puzzles, principles, and discoveries

Statement of inquiry

Investigating the relationship between quadratic functions and their models through representation and systems using scientific and technical innovations can lead to deeper understanding of their behavior and applications.

Inquiry questions

Factual—

- How do I graph a quadratic equation using technology?
- How do I use the Quadratic Formula to solve a quadratic?
- Where do I locate the x-intercepts on a graph?
- What are the steps in Completing the Square?

Conceptual—

- What does it mean to be a transformation of a quadratic equation?
- How can you determine that the Quadratic Formula will be the best method to solve a quadratic equation?

Debatable-

• What is the best method to use when solving a Quadratic Equation?

MYP Objectives	Assessment Tasks		
What specific MYP <u>objectives</u> will be addressed during this unit?	Relationship between summative assessment task(s) and statement of inquiry:	List of common formative and summative assessments.	
MYP A - Factoring MYP B - Dif.Of.Squares	Summative assessments will ask knowledge based questions on how to solve and model quadratic functions.	Formative Assessment(s): MYP A - Factoring MYP B - D.O.S Summative Assessment(s): Summative Part A over PAR.6 Summative Part B over FGR.7	
Approaches to learning (ATL)			

Approaches to learning (ATL)

Category: Thinking Skills
Cluster: Critical-thinking

Skill Indicator: Practice Observing carefully in order to recognize problems

Learning Experience: Beyond the Factoring

Category: Self-Management Skills

Cluster: Affective

Skill Indicator: Demonstrate persistence and perseverance Learning Experience: Protein Bar Toss (Diagnostic and Part 1) Category: Research Skills
Cluster: Information Literacy

Skill Indicator: Understand and use technology systems

Learning Experience: Graphing Transformations

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Learn	ııng	Expe	erier	ices

Add additional rows below as needed.

Objective or Content	Learning Experiences	Personalized Learning and Differentiation
A.PAR.6.2 - Fluently choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the expression. A.PAR.6.3 - Create and solve quadratic equations in one variable and explain the solution in the context of real-life phenomena. A.FGR.7.5 - Rewrite a quadratic function representing a contextual, mathematical to reveal the maximum or minimum value of the function it defines. Explain what the value describes in context. A.FGR.7.8 - Write a function defined by a quadratic expression in different but equivalent forms to reveal and explain different properties of the function.	Description: In this learning task, students will explore how to apply the quadratic formula to solving quadratic equations. Students will apply quadratic formulas to this particular equation and then practice using the quadratic formula with a variety of equations including some that could be solved by factoring or extraction of square roots. Students will apply the quadratic formula to solve a few quadratic equations with some irrational coefficients. Learning Goals: I can complete the square. I can use the quadratic formula to solve quadratic equations. I can use quadratic equations to solve problems. I can explore quadratic equations leading to irrational solutions.	Provide students with different assigned equations by groups
A.PAR.6.2 Fluently choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the expression. A.FGR.7.5 Rewrite a quadratic function representing a mathematically applicable situation to reveal the maximum or minimum value of the function it defines. Explain what the value describes in context. A.FGR.7.8 - Write a function defined by a quadratic expression in different but equivalent forms to reveal and explain different properties of the function.	Protein Bar Toss (Part 1) Description: In this learning plan, students will use factoring and completing the square of quadratic equations in the form ax2 + bx + c = 0 to solve problems. Students will analyze situations to determine which process will allow them to reach their solution. Students will use factoring and completing the square to identify and interpret key features of functions. The context is analysis of a situation where an object is tossed up and allowed to fall subject to gravity. In working with this context, students learn to interpret the intercepts of such functions and reinforce the concept that solving an equation of the form f(x) = 0 yields the x−intercepts of the graph of the function. Learning Goals: I can find maximums and minimums of quadratic functions. I can interpret the constant term in a quadratic equation in context. I can factor quadratic trinomials with a > 1. I can find and interpret the zeros, or x-intercepts, of a function.	Language Supports: The teacher will use the Quadratic equation -16t 2 + 24t + 160 to emphasize vocabulary such as "coefficient and constant". Supporting the Learning: The teacher will support students by using a highlighter to identify the A, B, and C value of a quadratic function. Extending the Learning: Ask students to use Desmos to graph 2-3 equations in the "Apply" part of the task after factoring. Students will be asked to create a real-world scenario that might apply to the graphs chosen.

A.FGR.7.2 Identify the effect on the graph generated by a quadratic function when replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs **A.FGR.7.3** Graph and analyze the key characteristics of quadratic functions.

A.FGR.7.4 Relate the domain and range of a quadratic function to its graph and, where applicable, to the quantitative relationship it describes.

Graphing Transformations

Description: In this learning plan, students will explore the transformations of the graph of $f(x) = x^2$. Students will use graphing technology to graph various parabolas and examine the changes from the parent function. Students will form conjectures about how kf(x), f(kx), f(x) + k, -f(x), and f(x + k) affect the graph of a quadratic function. They will then apply this new thinking to graph parabolas by hand and to describe a contextual situation.

Learning Goals:

- I can graph parabolas by hand and with graphing technology.
- I can investigate, identify, and explain transformations of quadratic functions.

Supporting the learning: The teacher will anticipate what students might struggle with during a lesson such as a reflection across the y intercept instead of the x intercept or shifting the equation to the right if the sign is positive and to the left if the sign is negative. Supporting the learning: The teacher will anticipate what students might struggle with during a lesson such as a reflection across the y intercept instead of the x intercept or shifting the equation to the right if the sign is positive and to the left if the sign is negative. Extending the Learning: After students have worked to complete the performance task, The teacher will ask students to use desmos.com to strengthen their understanding of Quadratic Equations and their transformations.

Content Resources

Textbook Correlation: enVision A | G | A - Algebra 1

A.PAR.6.1 - Lesson 7-4, 7-5, 7-6, 7-7

A.PAR.6.2 - Lesson 7-4, 7-5, 7-6, 7-7, 9-5

A.PAR.6.3 - Lesson 9-1, 9-2, 9-4, 9-5, 9-6, 9-7, Topic 9 - Math Modeling in 3 Acts

A.PAR.6.4 - Lesson 9-4, 9-5, 9-6, 9-7, Topic 9 - Math Modeling in 3 Acts

A.FGR.7.1 - Lesson 8-1, 8-2, 8-3, 8-4

A.FGR.7.2 - Lesson 8-1, 8-2, 8-3, 10-4, 10-5

A.FGR.7.3 - Lesson 8-1, 8-2, 8-3, Topic 8 - Math Modeling in 3 Acts

A.FGR.7.4 - Lesson 8-1. 8-4

A.FGR.7.5 - Lesson 8-3

A.FGR.7.6 - Lesson 8-4

A.FGR.7.7 - Lesson 8-5

A.FGR.7.8 - Lesson 8-3. 8-4

A.FGR.7.9 - Lesson 8-3, 8-5